**<u>Hoofdstuk 2: Functies en grafieken.</u>**

## 2.1 Lineaire functies

**<u>Opgave 1:</u>**
2 is de richtingscoëfficiënt, d.w.z. 1 naar rechts en 2 omhoog.
3 is het snijpunt met de $y$-as, dus $(0,3)$.

**<u>Opgave 2:</u>**
$rc = -\frac{1}{2}$ en het snijpunt met de $y$-as is $(0,2)$ dus $y = -\frac{1}{2}x + 2$

**<u>Opgave 3:</u>**
a. $y = -2x + b$ door $(-2,3)$
$3 = 4 + b$
$b = -1$
$l: \quad y = -2x - 1$

b. $rc_k = rc_m = 4$
$y = 4x + b$ door $(-5,21)$
$21 = -20 + b$
$b = 41$
$k: \quad y = 4x + 41$

**<u>Opgave 4:</u>**
a. $rc_p = rc_q = -\frac{1}{3}$
$y = -\frac{1}{3}x + b$ door $(-18,30)$
$30 = 6 + b$
$b = 24$
$p: \quad y = -\frac{1}{3}x + 24$

b. snijpunt $x$-as: $-\frac{1}{3}x + 24 = 0$
$-\frac{1}{3}x = -24$
$x = 72$ dus $(72,0)$
snijpunt $y$-as: $(0,24)$

**<u>Opgave 5:</u>**
a. $0 = -20a + 10$
$20a = 10$
$a = \frac{1}{2}$

b. $-4 = 2a + 10$
$-2a = 14$
$a = -7$

c. nee, want als $x = 0$ dan geldt $y = 10$ voor iedere waarde van $a$.

**<u>Opgave 6:</u>**
a. $0 = 16 + b$
$b = -16$

b. $rc_l = rc_m = -2$ dus $a = -2$
$7 = -20 + b$
$b = 27$

c. $l:\quad 6 = 8a - 4$
$-8a = -10$
$a = 1\frac{1}{4}$
$m:\quad 6 = -16 + b$
$b = 22$

d. $k:\quad 0{,}5x + 2 = 0$
$0{,}5x = -2$
$x = -4$ dus $(-4{,}0)$
$l:\quad 0 = -4a - 4$
$4a = -4$
$a = -1$
$m:\quad 0 = 8 + b$
$b = -8$

**Opgave 7:**

a. ga je 1 naar rechts, dan ga je $\frac{3}{4}$ omhoog, dus $rc_l = \frac{3}{4}$

b. $rc_l = \dfrac{y_B - y_A}{x_B - x_A} = \dfrac{3}{4}$

**Opgave 8:**

a. $rc = \dfrac{\Delta y}{\Delta x} = \dfrac{4-1}{1--1} = 1\frac{1}{2}$
$y = 1\frac{1}{2}x + b$ door $(-1{,}1)$
$1 = -1\frac{1}{2} + b$
$b = 2\frac{1}{2}$
$l:\quad y = 1\frac{1}{2}x + 2\frac{1}{2}$

b. $rc = \dfrac{\Delta y}{\Delta x} = \dfrac{0-5}{2--3} = -1$
$y = -x + b$ door $(-3{,}5)$
$5 = 3 + b$
$b = 2$
$k:\quad y = -x + 2$

c. $rc = \dfrac{\Delta y}{\Delta x} = \dfrac{3-3}{-7-5} = 0$
$y = b$ door $(5{,}3)$
$b = 3$
$m:\quad y = 3$

d. $rc = \dfrac{\Delta y}{\Delta x} = \dfrac{250-360}{160-180} = 5\frac{1}{2}$
$y = 5\frac{1}{2}x + b$ door $(180{,}360)$
$360 = 990 + b$

$b = -630$
$n: \quad y = 5\frac{1}{2}x - 630$

**Opgave 9:**

a. $rc = \frac{\Delta A}{\Delta s} = \frac{750-300}{21-15} = 75$

$A = 75s + b$ door $(15{,}300)$
$300 = 1125 + b$
$b = -825$
$A = 75s - 825$

b. $rc = \frac{\Delta R}{\Delta t} = \frac{35-10}{60-35} = 1$

$R = t + b$ door $(35{,}10)$
$10 = 35 + b$
$b = -25$
$R = t - 25$

**Opgave 10:**

a. $rc = \frac{\Delta p}{\Delta q} = \frac{2{,}25-7{,}75}{425-150} = -0{,}02$

$p = -0{,}02q + b$ door $(150;7{,}75)$
$7{,}75 = -3 + b$
$b = 10{,}75$
$p = -0{,}02q + 10{,}75$

b. $0{,}02q = -p + 10{,}75$
$q = -50p + 537{,}5$

c. gebruik de formule van opgave a: $p = -{,}02 \cdot 250 + 10{,}75 = 5{,}75$

d. gebruik de formule van opgave b: $q = -50 \cdot 4{,}25 + 537{,}5 = 325$

**Opgave 11:**

a. $rc = \frac{\Delta h}{\Delta t} = \frac{152{,}8-164{,}0}{18-12} = -\frac{11{,}2}{6} = -\frac{28}{15}$

$h = -\frac{28}{15}t + b$ door $(12{,}164)$
$164 = -22{,}4 + b$
$b = 186{,}4$
$h = -\frac{28}{15}t + 186{,}4$

b. $-\frac{28}{15}t + 186{,}4 = 156{,}7$
$-\frac{28}{15}t = -29{,}7$
$t = 15{,}91$
dus 14.15 uur en 55 sec.

**Opgave 12:**

a. $rc = \frac{\Delta L}{\Delta t} = \frac{175-167}{96-32} = \frac{8}{64} = 0{,}125$

$L = 0{,}125t + b$ door $(32{,}167)$

$167 = 4 + b$

$b = 163$

$L = 0{,}125t + 163$

b. $0{,}125t + 163 = 180$

$0{,}125t = 17$

$t = 136$ dus in 2036

**Opgave 13:**

a. $rc = \frac{\Delta B}{\Delta g} = \frac{1701{,}48 - 1381{,}90}{2906 - 2355} = 0{,}58$

$B = 0{,}58g + b$ door $(2355;1381{,}90)$

$1381{,}90 = 1365{,}9 + b$

$b = 16$

$B = 0{,}58g + 16$

b. vastrecht: € 16,-.

prijs per m³ gas: € 0,58.

c. $B = 0{,}58 \cdot 2281 + 16 = 1338{,}98$

## 2.2 Tweede- en derdegraadsfuncties

**Opgave 14:**

a. zie de figuur hiernaast.

b. (2,1)

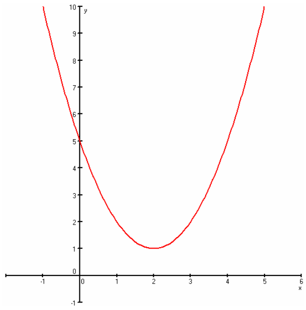
c. $y_1 = x^2 - 4x + 5$

tableset: tblstart=1

$\Delta$tbl=0,1

$y_1$ neemt af tot aan $x = 2$ en daarna neemt $y_1$ toe.

Dus het punt (2,1) is het laagste punt van de grafiek van $f$.



**Opgave 15:**

a. $y_1 = \frac{2}{3}x^3 - x^2 - 4x + \frac{2}{3}$

calc-menu optie maximum: $x = -1 \quad \wedge \quad y_1 = 3$

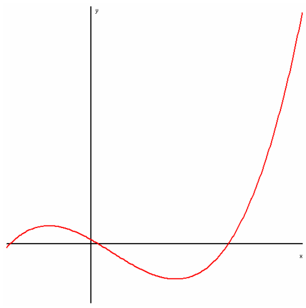
calc-menu optie minimum: $x = 2 \quad \wedge \quad y_1 = -6$

dus max $f(-1) = 3$ en min $f(2) = -6$

b. kleinste functiewaarde: $f(2) = -6$

grootste functiewaarde: $f(5) = 39$

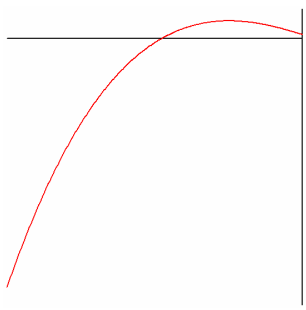
$B_f = [-6,39]$



c. kleinste functiewaarde: $f(-4) = -42$

grootste functiewaarde: $f(-1) = 3$

$B_f = [-42,3]$

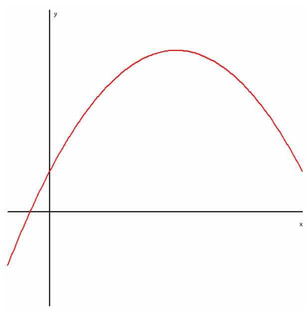


**Opgave 16:**

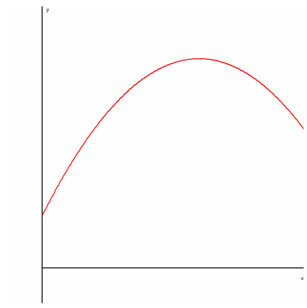
a. kleinste functiewaarde: $f(-1) = -4$

grootste functiewaarde: $f(3) = 12$

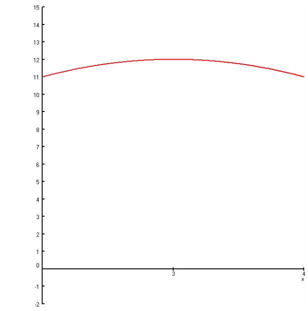
$B_f = [-4,12]$



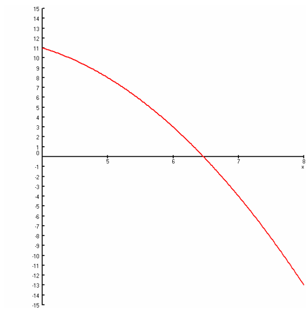
b. kleinste functiewaarde: $f(0) = 3$
grootste functiewaarde: $f(3) = 12$
$B_f = [3,12]$



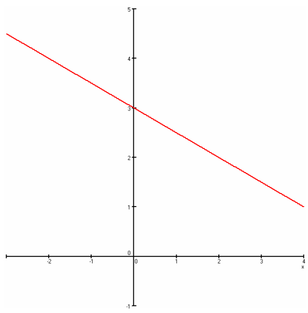
c. kleinste functiewaarde: $f(2) = f(4) = 11$
grootste functiewaarde: $f(3) = 12$
$B_f = [11,12]$



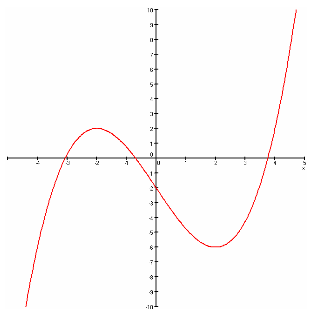
d. kleinste functie waarde: $f(8) = -13$
grootste functiewaarde: $f(4) = 11$
$B_f = [-13,11]$



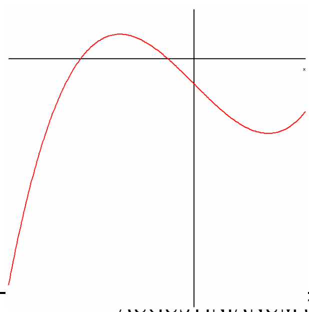
**<u>Opgave 17:</u>**
kleinste functiewaarde: $f(4) = 1$
grootste functiewaarde: $f(-3) = 4\frac{1}{2}$
$B_f = [1,4\frac{1}{2}]$



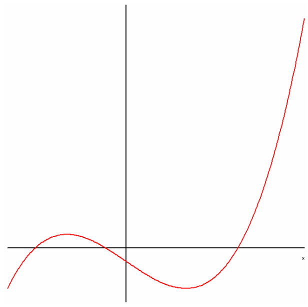
**<u>Opgave 18:</u>**
a. $y_1 = \frac{1}{4}x^3 - 3x - 2$
calc-menu optie maximum: $x = -2 \quad \wedge \quad y_1 = 2$
calc-menu optie minimum: $x = 2 \quad \wedge \quad y_1 = -6$
max $f(-2) = 2$ en min $f(2) = -6$



b. kleinste functiewaarde: $f(-5) = -18{,}25$
grootste functiewaarde: $f(-2) = 2$
$B_f = [-18{,}25;2]$



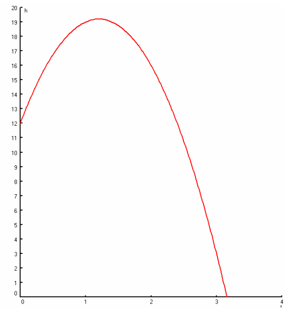
c. kleinste functiewaarde: $f(-4) = f(2) = -6$
grootste functiewaarde: $f(6) = 34$
$B_f = [-6,34]$



## Opgave 19:

a. $y_1 = -5x^2 + 12x + 12$
calc-menu optie zero: $x = 3{,}16$
$D_h = [0;3{,}2]$

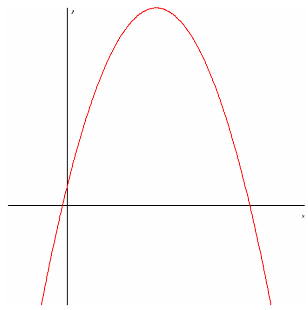
b. calc-menu optie maximum: $x = 1{,}2 \quad \wedge \quad y_1 = 19{,}2$
$B_h = [0;19{,}2]$



## Opgave 20:

a. $y_1 = -x^2 + 6x + 1$
calc-menu optie maximum: $x = 3 \quad \wedge \quad y_1 = 10$
top $(3{,}10)$

b. verschuif de grafiek van opgave a 10 naar beneden
dus $y_1 = -x^2 + 6x - 9$ dus $p = -9$

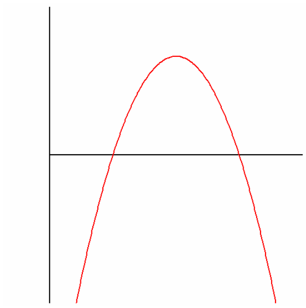


## Opgave 21:

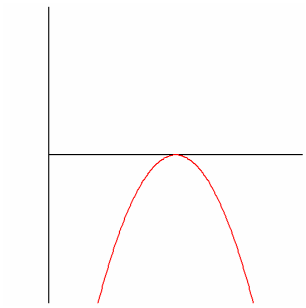
a. $D = (-6)^2 - 4 \cdot 2 \cdot p = 36 - 8p = 0$
$-8p = -36$
$p = 4\frac{1}{2}$

b. $D = 36 - 8p > 0$
$-8p > -36$
$p < 4\frac{1}{2}$

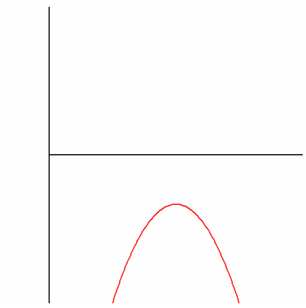
## Opgave 22:



2 snijpunten
$D > 0$



1 snijpunt
$D = 0$



geen snijpunten
$D < 0$

**Opgave 23:**

a. $D = (-5)^2 - 4 \cdot -\tfrac{1}{2} \cdot p = 25 + 2p = 0$

$2p = -25$

$p = -12\tfrac{1}{2}$

b. $D = 25 + 2p < 0$

$2p < -25$

$p < -12\tfrac{1}{2}$

**Opgave 24:**

a. $D = p^2 - 4 \cdot 3 \cdot 3 = p^2 - 36 = 0$

$p^2 = 36$

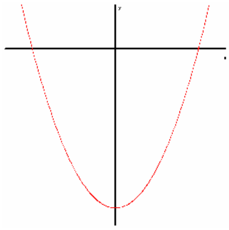
$p = 6 \quad \vee \quad p = -6$

b. dalparabool waarvan de top onder de $x$-as ligt, dus twee snijputen met de $x$-as.

$D = p^2 - 36 > 0$

$p = 6 \quad \vee \quad p = -6$

$p < -6 \quad \vee \quad p > 6$



**Opgave 25:**

a. minimum, dus de grafiek van $f_p$ is een dalparabool, dus $p > 0$

negatief minimum, dus de top ligt onder de $x$-as, dus de grafiek heeft twee snijpunten met de $x$-as, dus $D > 0$

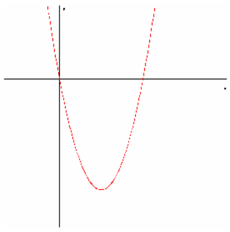
$D = (2p)^2 - 4 \cdot p \cdot 3 = 4p^2 - 12p > 0$

$4p(p-3) = 0$

$p = 0 \quad \vee \quad p = 3$

$p < 0 \quad \vee \quad p > 3$

dus $p > 3$



b. maximum, dus de grafiek van $f_p$ is een bergparabool, dus $p < 0$

negatief maximum, dus de top ligt onder de $x$-as, dus de grafiek heeft geen snijpunten met de $x$-as, dus $D < 0$

$0 < p < 3$

dus $p < 0 \quad \wedge \quad 0 < p < 3$

dus voor geen enkele waarde van $p$ heeft de grafiek van $f_p$ een negatief maximum.

**Opgave 26:**

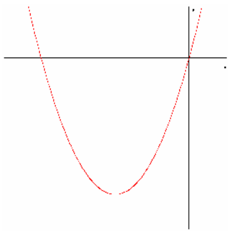
a. negatief maximum, dus de top ligt onder de $x$-as, dus de grafiek van $f_p$ heeft geen snijpunten met de $x$-as, dus $D < 0$.

$D = p^2 - 4 \cdot -1 \cdot 2p = p^2 + 8p < 0$

$p(p+8) = 0$

$p = 0 \quad \vee \quad p = -8$

$-8 < p < 0$



b. $f_p(2p) = -4$

$-(2p)^2 + p \cdot 2p + 2p = -4$

$-4p^2 + 2p^2 + 2p = -4$

$-2p^2 + 2p + 4 = 0$

$p^2 - p - 2 = 0$

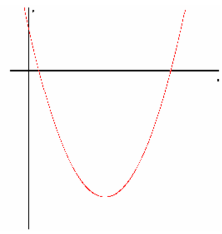
$(p+1)(p-2) = 0$

$p = -1 \quad \vee \quad p = 2$

**<u>Opgave 27:</u>**

a. minimum, dus de grafiek van $f_p$ is een dalparabool, dus $p > 0$

negatief minimum, dus de top ligt onder de $x$-as, dus de grafiek van $f_p$ heeft twee snijpunten met de $x$-as, dus $D > 0$



$D = (p+2)^2 - 4 \cdot p \cdot 3 > 0$

$p^2 + 4p + 4 - 12p > 0$

$p^2 - 8p + 4 > 0$

$p = \dfrac{8 \pm \sqrt{64-16}}{2} = \dfrac{8 \pm \sqrt{48}}{2}$

$p < \dfrac{8-\sqrt{48}}{2} \quad \vee \quad p > \dfrac{8+\sqrt{48}}{2}$

dus omdat $p > 0$ moet gelden: $0 < p < \dfrac{8-\sqrt{48}}{2} \quad \vee \quad p > \dfrac{8+\sqrt{48}}{2}$

b. maximum, dus de grafiek is een bergparabool, dus $p < 0$

positief maximum, dus de top ligt boven de $x$-as, dus de grafiek van $f_p$ heeft twee snijpunten met de $x$-as, dus $D > 0$

$\left( p < \dfrac{8-\sqrt{48}}{2} \quad \vee \quad p > \dfrac{8+\sqrt{48}}{2} \right) \quad \wedge \quad (p < 0)$

dus $p < 0$

**<u>Opgave 28:</u>**

a. $ax^2 + bx = 0$

$x(ax + b) = 0$

$x = 0 \quad \vee \quad ax = -b$

$x = 0 \quad \vee \quad x = -\dfrac{b}{a}$

dus de twee snijpunten met de $x$-as zijn voor $x = 0 \quad \vee \quad x = -\dfrac{b}{a}$

de top ligt precies midden tussen deze twee snijpunten, dus $x_{top} = \dfrac{0 + -\frac{b}{a}}{2} = -\frac{b}{2a}$

b. $+c$ betekent dat de grafiek van opgave a met $c$ omhoog wordt verschoven.
De $x$-coördinaat van de top verandert niet door deze verticale verschuiving.

**Opgave 29:**

$x_{top} = \frac{-p}{-4} = \frac{1}{4}p$

$y_{top} = -2(\frac{1}{4}p)^2 + p \cdot \frac{1}{4}p + 1$

$-\frac{1}{8}p^2 + \frac{1}{4}p^2 + 1 = 9$

$\frac{1}{8}p^2 = 8$

$p^2 = 64$

$p = 8 \quad \vee \quad p = -8$

**Opgave 30:**

$x_{top} = \frac{-p}{2} = -\frac{1}{2}p$

$y_{top} = (-\frac{1}{2}p)^2 + p \cdot -\frac{1}{2}p + 3 = \frac{1}{4}p^2 - \frac{1}{2}p^2 + 3 = -\frac{1}{4}p^2 + 3$

De top ligt op de lijn $y = x + 1$ dus $y_{top} = x_{top} + 1$

$-\frac{1}{4}p^2 + 3 = -\frac{1}{2}p + 1$

$-\frac{1}{4}p^2 + \frac{1}{2}p + 2 = 0$

$p^2 - 2p - 8 = 0$

$(p+2)(p-4) = 0$

$p = -2 \quad \vee \quad p = 4$

**Opgave 31:**

a. $x_{top} = \frac{-6}{2p} = -\frac{3}{p}$

$y_{top} = p \cdot (-\frac{3}{p})^2 + 6 \cdot -\frac{3}{p} + 1 = p \cdot \frac{9}{p^2} - \frac{18}{p} + 1 = \frac{9}{p} - \frac{18}{p} + 1 = -\frac{9}{p} + 1 = -2$

$-\frac{9}{p} = -3$

$p = 3$

b. $p = 3$ dus de grafiek is een dalparabool, dus de extreme waarde is een minimum.

**Opgave 32:**

$x_{top} = -\frac{p+2}{2p} = \frac{-p-2}{2p} = \frac{-p}{2p} - \frac{2}{2p} = -\frac{1}{2} - \frac{1}{p}$

$y_{top} = p \cdot (-\frac{1}{2} - \frac{1}{p})^2 + (p+2) \cdot (-\frac{1}{2} - \frac{1}{p}) + 5$

$= p \cdot (\frac{1}{4} + \frac{1}{p} + \frac{1}{p^2}) + -\frac{1}{2}p - 1 - 1 - \frac{2}{p} + 5$

$= \frac{1}{4}p + 1 + \frac{1}{p} - \frac{1}{2}p - \frac{2}{p} + 3$

$y_{top} = -\frac{1}{4}p - \frac{1}{p} + 4 = 3$

$-\frac{1}{4}p - \frac{1}{p} + 1 = 0$

$-\frac{1}{4}p^2 - 1 + p = 0$

$p^2 - 4p + 4 = 0$

$(p-2)^2 = 0$

$p = 2$

$x_{top} = -\frac{1}{2} - \frac{1}{2} = -1$

## Opgave 33:

$$x_{top} = \frac{-(p-4)}{2p} = \frac{-p+4}{2p} = \frac{-p}{2p} + \frac{4}{2p} = -\frac{1}{2} + \frac{2}{p}$$

$$\begin{aligned} y_{top} &= p \cdot (-\tfrac{1}{2} + \tfrac{2}{p})^2 + (p-4)(-\tfrac{1}{2} + \tfrac{2}{p}) + 3 \\ &= p \cdot (\tfrac{1}{4} - \tfrac{2}{p} + \tfrac{4}{p^2}) - \tfrac{1}{2}p + 2 + 2 - \tfrac{8}{p} + 3 \\ &= \tfrac{1}{4}p - 2 + \tfrac{4}{p} - \tfrac{1}{2}p + 7 - \tfrac{8}{p} \\ &= -\tfrac{1}{4}p - \tfrac{4}{p} + 5 \end{aligned}$$

De top ligt op de lijn $y = x + 9$ dus geldt: $y_{top} = x_{top} + 9$.

$-\frac{1}{4}p - \frac{4}{p} + 5 = -\frac{1}{2} + \frac{2}{p} + 9$

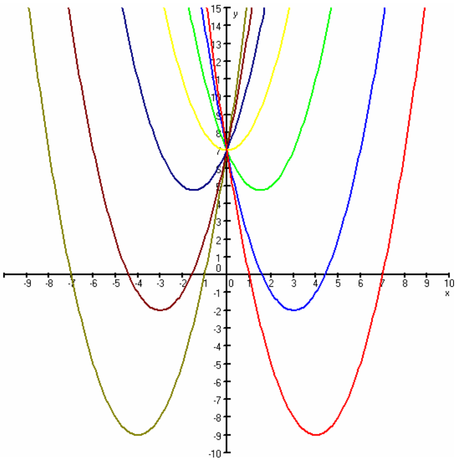
$y_1 = -\frac{1}{4}x - \frac{4}{x} + 5$ en $y_2 = \frac{2}{x} + 8\frac{1}{2}$

calc-menu optie intersection geeft: $x = -2 \quad \vee \quad x = -12$

dus $p = -2 \quad \vee \quad p = -12$

## Opgave 34:

a.

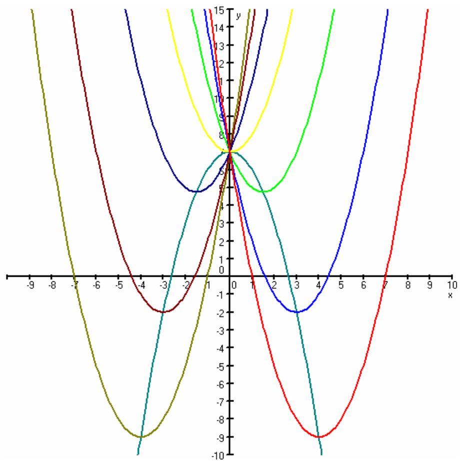


b. $x_{top} = \frac{-p}{2} = -\frac{1}{2}p$ dus $p = -2x_{top}$

$$\begin{aligned} y_{top} &= x^2 + -2x \cdot x + 7 \\ &= x^2 - 2x^2 + 7 \\ &= -x^2 + 7 \end{aligned}$$

dus alle toppen liggen op de parabool $y = -x^2 + 7$

c. zie de figuur hiernaast



## Opgave 35:

$x_{top} = \frac{-p}{-\frac{1}{4}} = 4p$ dus $p = \frac{1}{4}x_{top}$

$y_{top} = -\frac{1}{8}x^2 + \frac{1}{4}x \cdot x - 6 = -\frac{1}{8}x^2 + \frac{1}{4}x^2 - 6 = \frac{1}{8}x^2 - 6$

Dus alle toppen liggen op de kromme $y = \frac{1}{8}x^2 - 6$

**Opgave 36:**

$$x_{top} = \frac{-6}{2p} = -\tfrac{3}{p} \text{ dus } p = -\frac{3}{x_{top}}$$

$$y_{top} = -\frac{3}{x} \cdot x^2 + 6x + -\frac{3}{x} = -3x + 6x - \frac{3}{x} = 3x - \frac{3}{x}$$

Dus alle toppen liggen op de kromme $y = 3x - \dfrac{3}{x}$.

**Opgave 37:**

$$x_{top} = \frac{-p}{-2} = \tfrac{1}{2}p \text{ dus } p = 2x_{top}$$

$$y_{top} = -x^2 + 2x \cdot x + 2 \cdot 2x = -x^2 + 2x^2 + 4x = x^2 + 4x$$

Dus alle toppen liggen op de kromme $y = x^2 + 4x$.

**Opgave 38:**

$$x_{top} = \frac{2p}{2p^2} = \frac{1}{p} \text{ dus } p = \frac{1}{x_{top}}$$

$$y_{top} = (\tfrac{1}{x})^2 \cdot x^2 - 2 \cdot \tfrac{1}{x} \cdot x + 3 = \tfrac{1}{x^2} \cdot x^2 - 2 + 3 = 1 - 2 + 3 = 2$$

Dus alle toppen liggen op de kromme $y = 2$.

**Opgave 39:**

$$x_{top} = \frac{p}{2p} = \tfrac{1}{2}$$

Dus alle toppen liggen op de kromme $x = \tfrac{1}{2}$.

**Opgave 40:**

a. $x_{top} = \dfrac{-1}{2p}$

$$y_{top} = p \cdot (\tfrac{-1}{2p})^2 + \tfrac{-1}{2p} + \tfrac{1}{p}$$
$$= p \cdot \tfrac{1}{4p^2} - \tfrac{1}{2p} + \tfrac{1}{p}$$
$$= \tfrac{1}{4p} - \tfrac{1}{2p} + \tfrac{1}{p}$$
$$= \tfrac{1}{4p} - \tfrac{2}{4p} + \tfrac{4}{4p}$$
$$= \tfrac{3}{4p} = 6$$

$$24p = 3$$

$$p = \tfrac{1}{8}$$

b. $x_{top} = \dfrac{-1}{2p}$ dus $p = \dfrac{-1}{2x_{top}}$

$$y_{top} = \frac{3}{4p} = \frac{3}{4 \cdot \frac{-1}{2x_{top}}} = \frac{3}{\frac{-2}{x_{top}}} = -\tfrac{3}{2}x_{top}$$

Dus alle toppen liggen op de kromme $y = -1\tfrac{1}{2}x$.

**Opgave 41:**

a. $x_{top} = \frac{10}{2p} = \frac{5}{p}$

$y_{top} = p \cdot (\frac{5}{p})^2 - 10 \cdot \frac{5}{p} + p + 3$

$= p \cdot \frac{25}{p^2} - \frac{50}{p} + p + 3$

$= \frac{25}{p} - \frac{50}{p} + p + 3$

$= -\frac{25}{p} + p + 3$

De top ligt op de lijn $y = -x - 5$ dus geldt: $y_{top} = -x_{top} - 5$

$-\frac{25}{p} + p + 3 = -\frac{5}{p} - 5$

$-\frac{20}{p} + p + 8 = 0$

$-20 + p^2 + 8p = 0$

$p^2 + 8p - 20 = 0$

$(p+10)(p-2) = 0$

$p = -10 \quad \vee \quad p = 2$

als $p = -10$ geldt $x_{top} = \frac{5}{-10} = -\frac{1}{2}$ en $y_{top} = \frac{-25}{-10} - 10 + 3 = -\frac{1}{2}$ dus max $f(-\frac{1}{2}) = -4\frac{1}{2}$

als $p = 2$ geldt $x_{top} = \frac{5}{2}$ en $y_{top} = -\frac{25}{2} + 2 + 3 = -7\frac{1}{2}$ dus min $f(2\frac{1}{2}) = -7\frac{1}{2}$

b. $x_{top} = \frac{5}{p}$ dus $p = \dfrac{5}{x_{top}}$

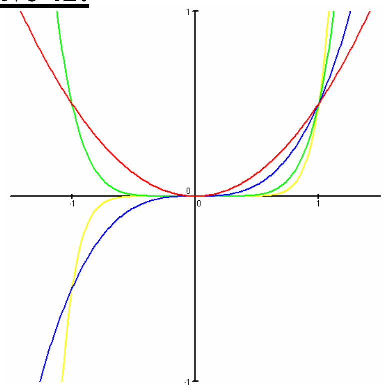
$y_{top} = -\frac{25}{p} + p + 3 = -\dfrac{25}{\frac{5}{x_{top}}} + \dfrac{5}{x_{top}} + 3 = -5x_{top} + \dfrac{5}{x_{top}} + 3$

Dus alle toppen liggen op de kromme $y = -5x + \frac{5}{x} + 3$.

## 2.3 Grafieken veranderen

**Opgave 42:**

a.



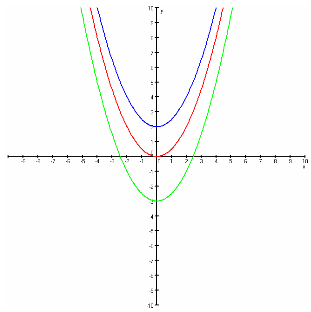
b. $(0,0)$ en $(1,\frac{1}{2})$

c. $y_1$ en $y_3$

d. $y_1$ en $y_3$

**Opgave 43:**

a.



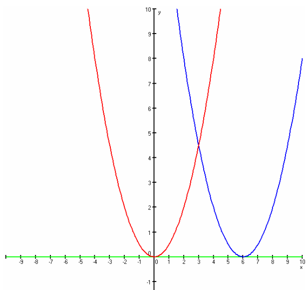
b. translatie over $(0,2)$

c. translatie over $(0,-3)$

d. als je de grafiek van $y = 0{,}5x^2$ transleert over $(0,6)$ krijgt je de grafiek van $y = 0{,}5x^2 + 6$.

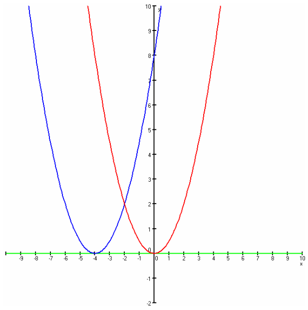
**Opgave 44:**

a.



transleer over $(6,0)$

b.



transleer over $(-4,0)$

c. als je de grafiek van $y = 0{,}5x^2$ transleert over $(2,0)$ krijg je de grafiek van $y = 0{,}5(x-2)^2$.

**Opgave 45:**

a. $y = -5(x-2)^2 + 5$

b. $y = -5(x+3)^2 + 6$

c. $y = -5(x-7)^2$

**Opgave 46:**

$g(x) = 2(x+2)^2$ $\quad h(x) = 2(x-2)^2 - 2$ $\quad k(x) = 2(x+1)^2 - 3$ $\quad l(x) = 2(x-1)^2 - 4$

**Opgave 47:**

a. max $f(0) = 2$ $\quad B_f = \langle \leftarrow, 2]$

b. max $g(2) = 8$ $\quad B_g = \langle \leftarrow, 8]$

c. min $h(-1) = 0$ $\quad B_h = [0, \rightarrow\rangle$

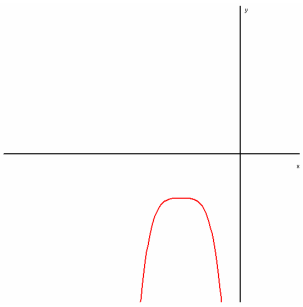
d. min $k(0) = 1$ $\quad B_k = [1, \rightarrow\rangle$

e. max $l(100) = 0$ $\quad B_l = \langle \leftarrow, 0]$

f. max $m(-0{,}1) = -0{,}3$ $\quad B_m = \langle \leftarrow; -0{,}3]$

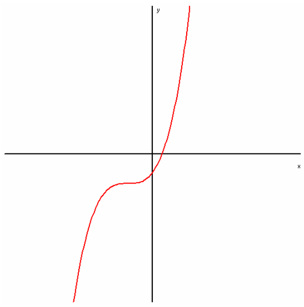
**Opgave 48:**

a.



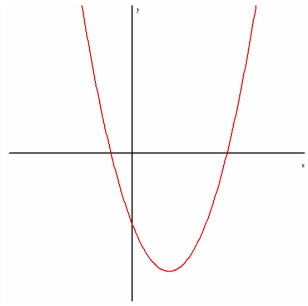
max $f(-2) = -3$

b.



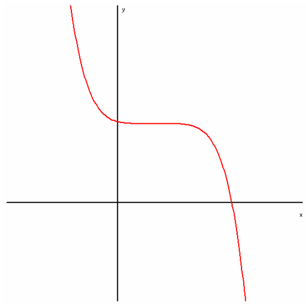
punt van symmetrie $(-\tfrac{1}{2}, -2)$

c.



min $h(3) = -4$

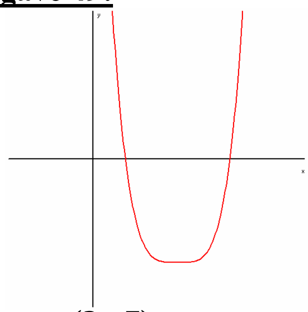
d.



punt van symmetrie $(1,4)$

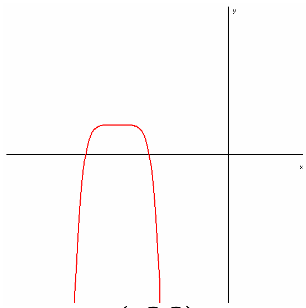
**Opgave 49:**

a.



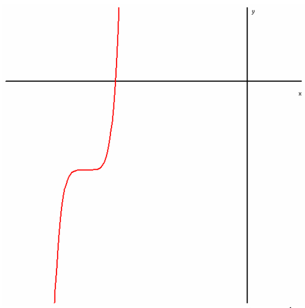
top $(2,-7)$

b.



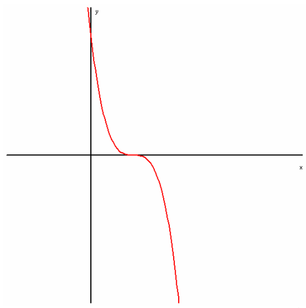
top $(-3,2)$

c.



punt van symmetrie $(-3,2)$

d.

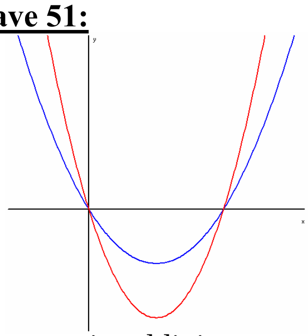


punt van symmetrie $(1,0)$

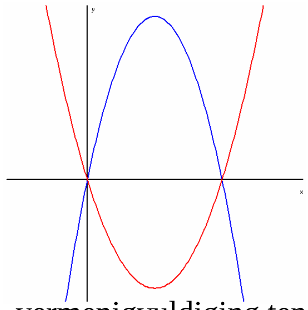
**Opgave 50:**

**Opgave 51:**

a.



vermenigvuldiging ten opzichte van de $x$-as met factor 0,5.

b. 

vermenigvuldiging ten opzichte van de $x$-as met factor $-1{,}5$.

**Opgave 52:**

$y=-0{,}5x^3 \xrightarrow{T(-3,-5)} y=-0{,}5(x+3)^3-5 \xrightarrow{V_{x-as},-3} y=-3\cdot(-{,}05(x+3)^3-5)$

dus $y=1{,}5(x+3)^3+15$

**Opgave 53:**

a. top $(3,7)$ $\xrightarrow{T(1,2)}$ top $(4,9)$ $\xrightarrow{V_{x-as},1\frac{1}{2}}$ top $(4,13\frac{1}{2})$

b. top $(-4,-7)$ $\xrightarrow{V_{x-as},2}$ top $(-4,-14)$ $\xrightarrow{T(-1,3)}$ top $(-5,-11)$

**Opgave 54:**

a. $y=0{,}3x^4 \xrightarrow{T(-5,6)} y=0{,}3(x+5)^4+6 \xrightarrow{V_{x-as},-3} y=-0{,}9(x+5)^4-18$

top $-5,-18)$

b. $y=0{,}3x^4 \xrightarrow{V_{x-as},-3} y=-0{,}9x^4 \xrightarrow{T(-5,6)} y=-0{,}9(x+5)^4+6$

top $(-5,6)$

**Opgave 55:**

a. vermenigvuldiging ten opzichte van de $x$-as met factor $-1$.

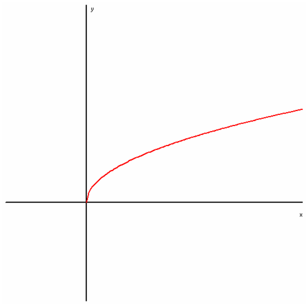
b. $y=-3(x-1)^2+6$

## 2.4 Wortelfuncties

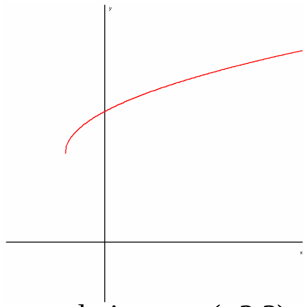
**Opgave 56:**

a. $D = [0, \rightarrow\rangle$

$B = [0, \rightarrow\rangle$



b.



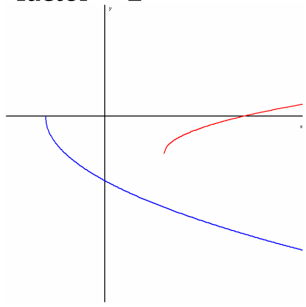
translatie over $(-2,3)$

**Opgave 57:**

a. $f$ translatie over $(3,-2)$

$g$ eerst translatie over $(-3,0)$, daarna vermenigvuldiging ten opzichte van de $x$-as met factor $-2$

b.



beginpunt $f$: $(3,-2)$

beginpunt $g$: $(-3,0)$

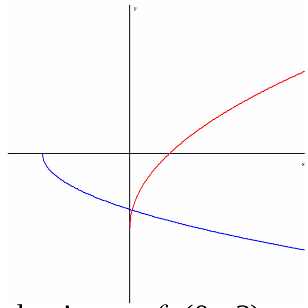
c. $D_f = [3, \rightarrow\rangle$ , $B_f = [-2, \rightarrow\rangle$ , $D_g = [-3, \rightarrow\rangle$ , $B_g = \langle \leftarrow, 0]$

**Opgave 58:**

a. $f$: eerst vermenigvuldiging ten opzichte van de $x$-as met factor 2 , daarna translatie over $(0,-3)$

$g$: eerst translatie over $(-5,0)$ daarna vermenigvuldiging ten opzichte van de $x$-as met factor $-1$

b.



beginpunt $f$ $(0,-3)$
beginpunt $g$ $(-5,0)$

c. $D_f = [0,\rightarrow\rangle$ , $B_f = [-3,\rightarrow\rangle$ , $D_g = [-5,\rightarrow\rangle$ , $B_g = \langle\leftarrow,0]$

**Opgave 59:**

a. beginpunt $(-5,3)$ $D_f = [-5,\rightarrow\rangle$ $B_f = [3,\rightarrow\rangle$
b. beginpunt $(-3,-7)$ $D_g = [-3,\rightarrow\rangle$ $B_g = [-7,\rightarrow\rangle$
c. beginpunt $(-1,0)$ $D_h = [-1,\rightarrow\rangle$ $B_h = \langle\leftarrow,0]$
d. beginpunt $(0,1)$ $D_k = [0,\rightarrow\rangle$ $B_k = [1,\rightarrow\rangle$
e. beginpunt $(1,-1)$ $D_l = [1,\rightarrow\rangle$ $B_l = \langle\leftarrow,-1]$
f. beginpunt $(0,-3)$ $D_m = [0,\rightarrow\rangle$ $B_m = [-3,\rightarrow\rangle$

**Opgave 60:**

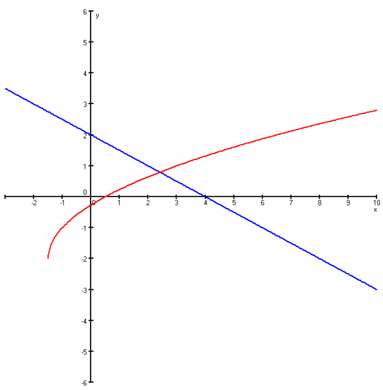
**Opgave 61:**

a. $(2,1)$
b. je komt niet precies op het beginpunt uit, dat komt omdat het rekenmachine in stapjes rekent die geen mooie getallen zijn

**Opgave 62:**

a.

| $x$ | $-1\frac{1}{2}$ | $-1$ | $\frac{1}{2}$ | 3 | $6\frac{1}{2}$ |
|---|---|---|---|---|---|
| y | $-2$ | $-1$ | 0 | 1 | 2 |



b. $B_f = [-2,\rightarrow\rangle$

c. $y_1 = -2 + \sqrt{2x+3}$
$y_2 = -0{,}5x + 2$
calc-menu de optie intersection
geeft $x = 2{,}41$
$-1\frac{1}{2} \leq x < 2{,}41$

**Opgave 63:**

a. $8 - 4x \geq 0$
$-4x \geq -8$
$x \leq 2$
$D_f = \langle\leftarrow,2]$ $B_f = [3,\rightarrow\rangle$ beginpunt $(2,3)$

b. $4x - 8 \geq 0$

$4x \geq 8$

$x \geq 2$

$D_g = [2, \rightarrow\rangle$ $\quad$ $B_g = [3, \rightarrow\rangle$ $\quad$ beginpunt $(2,3)$

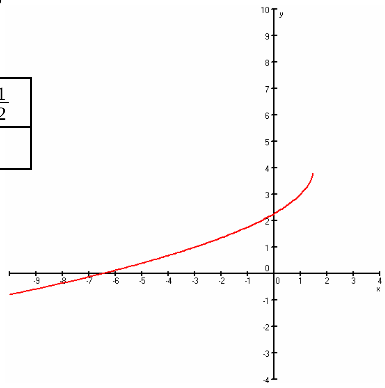
c. $2x + 6 \geq 0$

$2x \geq -6$

$x \geq -3$

$D_h = [-3, \rightarrow\rangle$ $\quad$ $B_h = \langle\leftarrow, 5]$ $\quad$ beginpunt $(-3,5)$

d. $D_k = [0, \rightarrow\rangle$ $\quad$ $B_k = \langle\leftarrow, 3]$ $\quad$ beginpunt $(0,3)$



**Opgave 64:**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | $-11$ | $-6\frac{1}{2}$ | $-3$ | $-\frac{1}{2}$ | $1$ | $1\frac{1}{2}$ |
| $y$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |

a. $3 - 2x \geq 0$

$-2x \geq -3$

$x \leq 1\frac{1}{2}$

$D_f = \langle\leftarrow, 1\frac{1}{2}]$

b. $B_f = \langle\leftarrow, 4]$

c. $4 - \sqrt{3 - 2x} = -1$

$-\sqrt{3 - 2x} = -5$

$\sqrt{3 - 2x} = 5$

$3 - 2x = 25$

$-2x = 22$

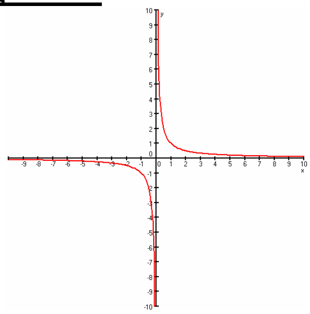
$x = -11$

$-11 < x \leq 1\frac{1}{2}$

## 2.5 Gebroken functies

**Opgave 65:**

a.



b. $f(x)$ komt steeds dichter bij 0 en blijft positief

$f(x)$ komt steeds dichter bij 0 en blijft negatief

c. $f(x)$ wordt steeds kleiner en blijft negatief

we zeggen: $f(x)$ gaat naar $-\infty$

d. delen door 0 kan niet, dus het rekenmachine geeft een foutmelding.

**Opgave 66:**

a. translatie over $(-2,-3)$

b. horizontale asymptoot H.A.: $y=-3$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | $-0{,}1$ | $-0{,}09$ | $-0{,}08$ | $-0{,}07$ | $-0{,}06$ | $-0{,}05$ | $-0{,}04$ | $-0{,}03$ | $-0{,}02$ | $-0{,}01$ |
| $y$ | $-10$ | $-11{,}1$ | $-12{,}5$ | $-14{,}3$ | $-16{,}7$ | $-20$ | $-25$ | $-33{,}3$ | $-50$ | $-100$ |

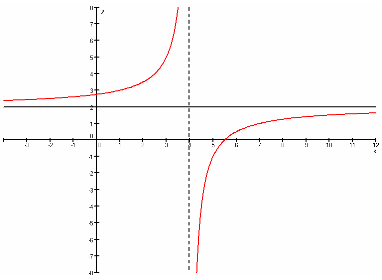
verticale asymptoot V.A.: $x=-2$

**Opgave 67:**

a. eerst vermenigvulding ten opzichte van de $x$-as met factor $-3$, daarna translatie over $(4,2)$

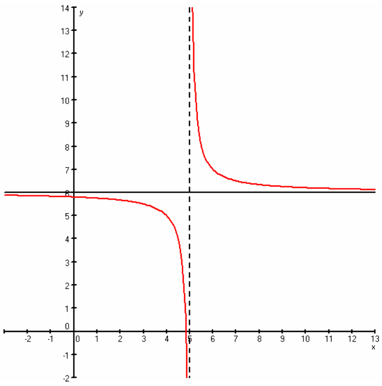
b. H.A.: $y=2$

V.A.: $x=4$



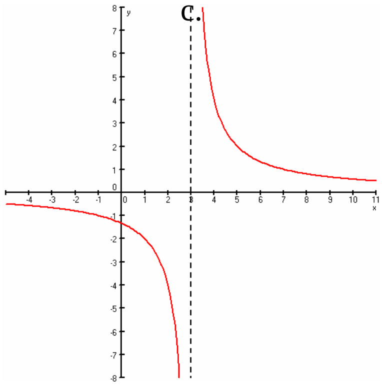
**Opgave 68:**

a.



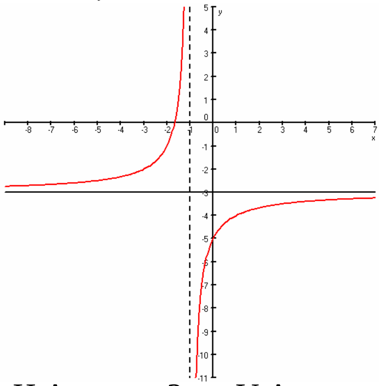
H.A.: $y=6$ V.A.: $x=5$

c.



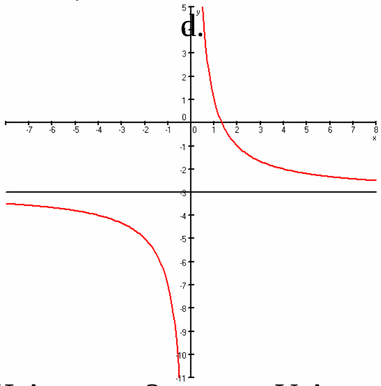
H.A.: $y=0$ V.A.: $x=3$

b.



H.A.: $y=-3$ V.A.: $x=-1$

d.

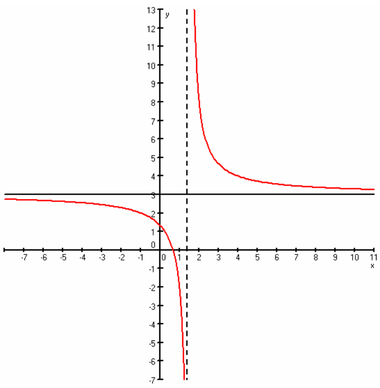


H.A.: $y=-3$ V.A.: $x=0$

**Opgave 69:**

**Opgave 70:**

a.



b.

| $x$ | 0 | 100 | 200 | 300 | 400 | 500 |
|---|---|---|---|---|---|---|
| $y$ | 1,33 | 3,025 | 3,013 | 3,008 | 3,006 | 3,005 |

$f(x)$ gaat naar 3, dus H.A.: $y=3$

c.

| $x$ | 1,4 | 1,41 | 1,42 | 1,43 | 1,44 | 1,45 | 1,46 | 1,47 | 1,48 | 1,49 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-22$ | $-24{,}8$ | $-28{,}3$ | $-32{,}7$ | $-38{,}7$ | $-47$ | $-59{,}5$ | $-80{,}3$ | $-122$ | $-247$ |

$f(x)$ gaat naar $-\infty$ dus V.A.: $x=1\frac{1}{2}$

## Opgave 71:

a. $noemer = 0$ geeft $x = 4$ dus V.A.: $x = 4$

voor grote $x$ is $f(x) \approx \dfrac{3x}{-x} = -3$ dus H.A.: $y = -3$

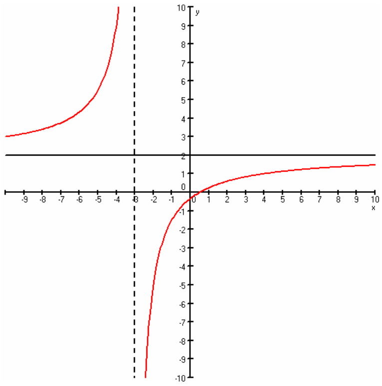
b. $noemer = 0$ geeft $5 + 2x = 0$

$$2x = -5$$

$$x = -2\tfrac{1}{2} \text{ dus V.A.: } x = -2\tfrac{1}{2}$$

voor grote $x$ is $g(x) \approx \dfrac{2x}{2x} = 1$ dus H.A.: $y = 1$

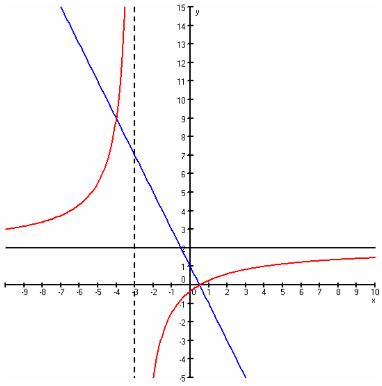
## Opgave 72:



a. $noemer = 0$ geeft $x = -3$ dus V.A.: $x = -3$

voor grote $x$ is $f(x) \approx \dfrac{2x}{x} = 2$ dus H.A.: $y = 2$

| $x$ | $-10$ | $-5$ | $-4$ | $-2$ | $0$ | $4$ | $10$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 3 | 5,5 | 9 | $-5$ | $-\frac{1}{3}$ | 1 | 1,46 |

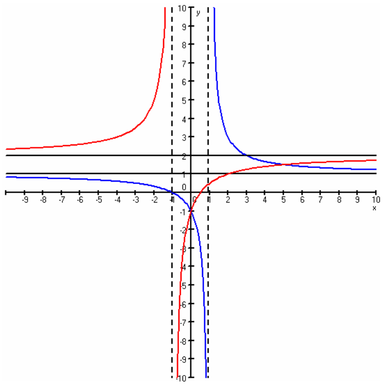


b. $y_1 = \dfrac{2x-1}{x+3}$ en $y_2 = -2x + 1$

calc-menu intersection geeft $x = -4 \quad \vee \quad x = \frac{1}{2}$

$x \leq -4 \vee \quad -3 < x \leq \frac{1}{2}$

## Opgave 73:



a. grafiek van *f*:

$noemer = 0$ geeft $x = -1$ dus V.A.: $x = -1$

voor grote $x$ is $f(x) \approx \dfrac{2x}{x} = 2$ dus H.A.: $y = 2$

grafiek van *g*:

$noemer = 0$ geeft $x = 1$ dus V.A.: $x = 1$

voor grote $x$ is $g(x) \approx \dfrac{x}{x} = 1$ dus H.A.: $y = 1$

b. $\dfrac{2x-1}{x+1} = \dfrac{x+1}{x-1}$

$(2x-1)(x-1) = (x+1)(x+1)$

$2x^2 - 2x - x + 1 = x^2 + x + x + 1$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad \vee \quad x = 5$

$-1 < x < 0 \quad \vee \quad x > 5$

c. $\frac{2x-1}{x+1}=4$ $\qquad$ $\frac{x+1}{x-1}=4$

$2x-1=4(x+1)$ $\qquad$ $x+1=4(x-1)$

$2x-1=4x+4$ $\qquad$ $x+1=4x-4$

$-2x=5$ $\qquad$ $-3x=-5$

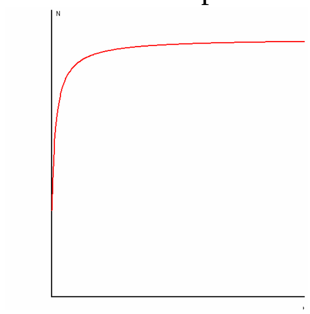
$x_A=-2\frac{1}{2}$ $\qquad$ $x_B=\frac{5}{3}$

$AB=x_B-x_A=\frac{5}{3}--2\frac{1}{2}=4\frac{1}{6}$

**Opgave 74:**

a. $N=1800$ dus op den duur zijn er 1800 insecten

b.



c. $y_1=1800-\frac{1200}{1+3x}$ en $y_2=1760$

calc-menu intersection geeft $x=9{,}67$ dus op de tiende dag

d. $N(4)-N(3)=1708-1680=28$

e. $N=1680$ voor $t=3$

$N=1745$ voor $t=7$

dus dat duurt 4 dagen

**Opgave 75:**

a. $\frac{1}{f}=\frac{1}{b}+\frac{1}{v}$

$\frac{1}{3}=\frac{1}{b}+\frac{1}{v}$

$-\frac{1}{b}=\frac{1}{v}-\frac{1}{3}$

$\frac{1}{b}=\frac{1}{3}-\frac{1}{v}$

$\frac{1}{b}=\frac{1}{3}-\frac{1}{v}=\frac{v}{3v}-\frac{3}{3v}=\frac{v-3}{3v}$

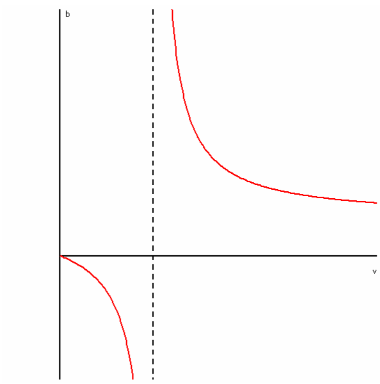
$b=\frac{3v}{v-3}$

b. $noemer=0$ geeft $v=3$ dus V.A.: $v=3$

als de voorwerpsafstand bijna 3 is, wordt de brandpuntsafstand plus of min oneindig

voor grote $v$ is $b\approx\frac{3v}{v}=3$ dus H.A.: $b=3$

als het voorwerp ver weg staat is de brandpuntsafstand 3



c. $b = v$

$$v = \frac{3v}{v-3}$$

$$v(v-3) = 3v$$

$$v^2 - 3v = 3v$$

$$v^2 - 6v = 0$$

$$v(v-6) = 0$$

$$v = 0 \text{ (vervalt)} \quad \vee \quad v = 6$$

d. $\left|\frac{b}{v}\right| = 2$

| | |
|---|---|
| $\frac{b}{v} = 2$ $\quad\vee$ | $\frac{b}{v} = -2$ |
| $b = 2v$ | $b = -2v$ |
| $2v = \frac{3v}{v-3}$ | $-2v = \frac{3v}{v-3}$ |
| $2v(v-3) = 3v$ | $-2v(v-3) = 3v$ |
| $2v^2 - 6v = 3v$ | $-2v^2 + 6v = 3v$ |
| $2v^2 - 9v = 0$ | $-2v^2 + 3v = 0$ |
| $2v(v - 4\tfrac{1}{2}) = 0$ | $-2v(v - 1\tfrac{1}{2}) = 0$ |

$$v = 0 \text{ (vervalt)} \quad \vee \quad v = 4\tfrac{1}{2} \quad \vee \quad v = 1\tfrac{1}{2}$$

## Diagnostische toets Hoofdstuk 2.

### Opgave 1:

a. $y = 2x + b$ door $(-1{,}6)$

$6 = -2 + b$

$b = 8$

$k: \quad y = 2x + 8$

b. $rc = rc_m = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$ door $(9{,}3)$

$3 = -4\frac{1}{2} + b$

$b = 7\frac{1}{2}$

$l: \quad y = -\frac{1}{2}x + 7\frac{1}{2}$

c. $0 = -10a + 5$

$10a = 5$

$a = \frac{1}{2}$

### Opgave 2:

a. $rc = \frac{\Delta y}{\Delta x} = \frac{-2-2}{3--5} = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$ door $(-5{,}2)$

$2 = 2\frac{1}{2} + b$

$b = -\frac{1}{2}$

$k: \quad y = -\frac{1}{2}x - \frac{1}{2}$

b. $rc = \frac{\Delta y}{\Delta x} = \frac{135-60}{65-40} = 3$

$y = 3x + b$ door $(40{,}60)$

$60 = 120 + b$

$b = -60$

$l: \quad y = 3x - 60$

### Opgave 3:

a. $rc = \frac{\Delta W}{\Delta t} = \frac{2900-500}{12-4} = 300$

$W = 300t + b$ door $(4{,}500)$

$500 = 1200 + b$

$b = -700$

$W = 300t - 700$

b. $W = 5{,}2 \cdot 300 - 700 = 860$

### Opgave 4:

a. $rc = \frac{\Delta A}{\Delta p} = \frac{665-800}{9{,}75-7{,}50} = -60$

$A = -60p + b$ door $(7{,}5;800)$

$800 = -450 + b$

$b = 1250$

$A = -60p + 1250$

b. $A = -60 \cdot 11{,}25 + 1250 = 575$

c. $-60p + 1250 > 1000$

$-60p > -250$

$p < 4{,}17$

**Opgave 5:**

a. $y_1 = 0{,}5x^3 - 6x^2 + 30$

calc-menu optie maximum geeft $x = 0 \quad \wedge \quad y = 30$

calc-menu optie minimum geeft $x = 8 \quad \wedge \quad y = -98$

dus max $f(0) = 30$ en min $f(8) = -98$

b. kleinste waarde: $f(-4) = -98$

grootste waarde: $f(0) = 30$

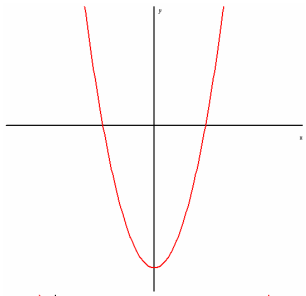
dus $B_f = [-98{,}30]$

c. kleinste waarde: $f(8) = -98$

grootste waarde: $f(13) = 114{,}5$

dus $B_f = [-98;114{,}5]$

**Opgave 6:**



a. $D = p^2 - 12 = 0$

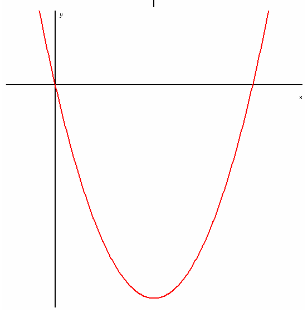
$p^2 = 12$

$p = \sqrt{12} \quad \vee \quad p = -\sqrt{12}$

b. $D = p^2 - 12 > 0$

$p < -\sqrt{12} \quad \vee \quad p > \sqrt{12}$

**Opgave 7:**



a. $D = p^2 - 24p < 0$

$p(p - 24) = 0$

$p = 0 \quad \vee \quad p = 24$

$0 < p < 24$

b. $p^2 + p^2 + 6p = -4$

$2p^2 + 6p + 4 = 0$

$p^2 + 3p + 2 = 0$

$(p+1)(p+2) = 0$

$p = -1 \quad \vee \quad p = -2$

c. $x_{top} = -\dfrac{p}{2} = -\tfrac{1}{2}p$

$y_{top} = (-\tfrac{1}{2}p)^2 + p \cdot -\tfrac{1}{2}p + 6p = \tfrac{1}{4}p^2 - \tfrac{1}{2}p^2 + 6p = -\tfrac{1}{4}p^2 + 6p = -13$

$-\tfrac{1}{4}p^2 + 6p + 13 = 0$

$p^2 - 24p - 52 = 0$

$(p+2)(p-26) = 0$

$p = -2 \quad \vee \quad p = 26$

d. $-\frac{1}{4}p^2+6p=2\cdot-\frac{1}{2}p+13$

$-\frac{1}{4}p^2+7p-13=0$

$p^2-28p+52=0$

$(p+2)(p\ 26)=0$

$p=2 \quad \vee \quad p=26$

**Opgave 8:**

a. $x_{top}=-\dfrac{2p}{2}=-p$ dus $p=-x_{top}$

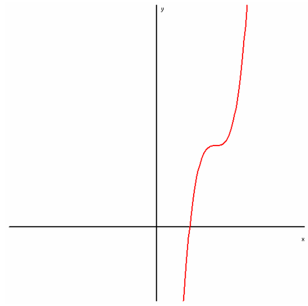
$y_{top}=(-p)^2+2p\cdot-p+p=p^2-2p^2+p=-p^2+p$

$y_{top}=-(-x_{top})^2+-x_{top}=-(x_{top})^2-x_{top}$

dus alle toppen liggen op de kromme $y=-x^2-x$

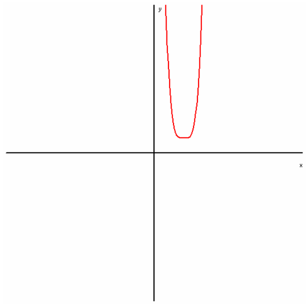
**Opgave 9:**

a.



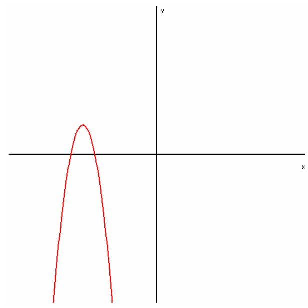
punt van symmetrie $(4,11)$

c.



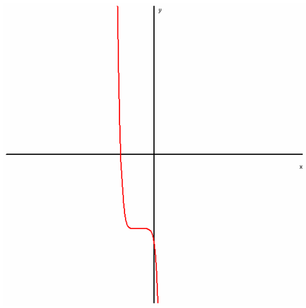
top $(2,1)$

b.



top $(-5,2)$

d.



punt van symmetrie $(-1,-5)$

**Opgave 10:**

$g(x)=2(x-3)^2+5 \quad B_g=[5,\rightarrow\rangle \qquad$ min $g(3)=5$

**Opgave 11:**

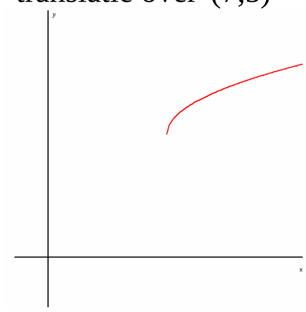
a. top $(2,2)$ $\xrightarrow{V_{x-as,3}}$ top $(2,6)$ $\xrightarrow{T(3,-4)}$ top $(5,2)$

b. top $(2,2)$ $\xrightarrow{T(3,-4)}$ top $(5,-2)$ $\xrightarrow{V_{x-as,3}}$ top $(5,-6)$

**Opgave 12:**

a. translatie over $(7{,}5)$

b.



beginpunt $(7{,}5)$

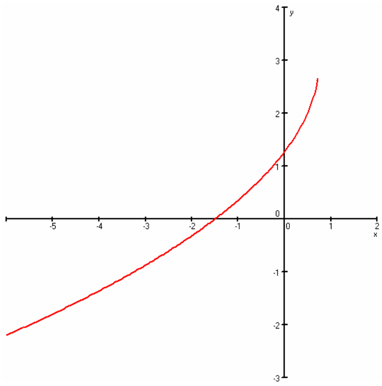
c. $D_f = [7, \rightarrow\rangle$ $\quad B_f = [5, \rightarrow\rangle$

**Opgave 13:**

a. $3-4x \geq 0$

$-4x \geq -3$

$x \leq -\frac{3}{4}$



| $x$ | $-3\frac{1}{4}$ | $-1\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ |
|---|---|---|---|---|---|
| $y$ | $-1$ | $0$ | $1$ | $2$ | $3$ |

b. $B_f = \langle \leftarrow, 3]$

c. $3-\sqrt{3-4x} = 1$

$-\sqrt{3-4x} = -2$

$3-4x = 4$

$-4x = 1$

$x = -\frac{1}{4}$

$x < -\frac{1}{4}$

d. $3-\sqrt{3-4x} = 1,$

$-\sqrt{3-4x} = -1\frac{1}{2}$

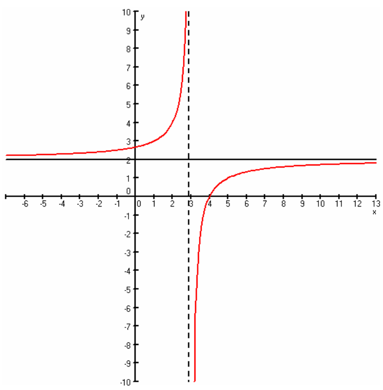
$3-4x = 2\frac{1}{4}$

$-4x = -\frac{3}{4}$

$x = \frac{3}{16}$

$\frac{3}{16} < x \leq \frac{3}{4}$

**Opgave 14:**



a. eerst vermenigvuldiging ten opzichte van de $x$-as met factor $-2$ en daarna translatie over $(3{,}2)$.

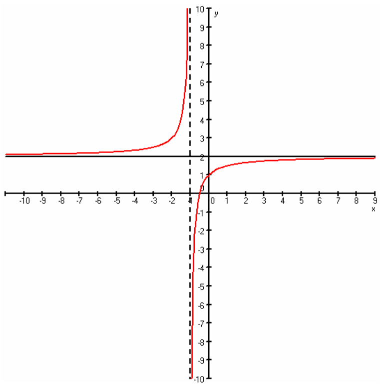
b. $noemer = 0$ geeft $x = 3$ dus V.A.: $x = 3$

voor grote $x$ is $f(x) \approx 0 + 2 = 2$ dus H.A.: $y = 2$

**Opgave 15:**

a. $noemer = 0$ geeft $x = -1$ dus V.A.: $x = -1$

voor grote $x$ is $f(x) \approx \frac{2x}{x} = 2$ dus H.A.: $y = 2$



b. $\frac{2x+1}{x+1} = -\frac{1}{2}x + 2$

$2x + 1 = (x+1)(-\frac{1}{2}x + 2)$

$2x + 1 = -\frac{1}{2}x^2 + 2x - \frac{1}{2}x + 2$

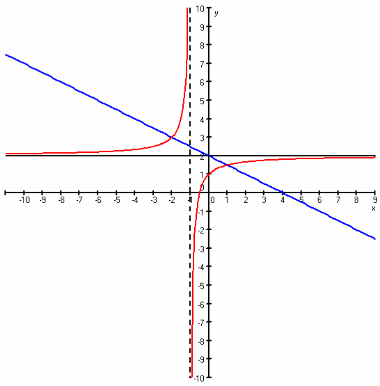
$\frac{1}{2}x^2 + \frac{1}{2}x - 1 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad \vee \quad x = 1$

$-2 \le x < -1 \quad \vee \quad x \ge 1$



## GEMENGDE OPGAVEN: H2 Functies en grafieken.

**Opgave 10:**

a. $rc_l = rc_m = -1\frac{1}{2}$ dus $a = -1\frac{1}{2}$

$y = -1\frac{1}{2}x + b$ door $(0,1)$

$1 = 0 + b$ dus $b = 1$

b. snijpunt met de $x$-as van lijn $k$:

$\frac{1}{2}x + 1 = 0$

$\frac{1}{2}x = -1$

$x = -2$

$l: \quad y = ax - 2$ door $(-2,0)$

$0 = -2a - 2$

$2a = -2$

$a = -1$

$m: \quad y = -1\frac{1}{2}x + b$ door $(-2,0)$

$0 = 3 + b$

$b = -3$

c. $l: \quad y = ax - 2$ door $(4,3)$

$3 = 4a - 2$

$-4a = -5$

$a = 1\frac{1}{4}$

$m: \quad y = -1\frac{1}{2}x + b$ door $(4,3)$

$3 = -6 + b$

$b = 9$

**Opgave 11:**

a. $rc = \dfrac{\Delta B}{\Delta x} = \dfrac{183{,}95 - 129{,}14}{265 - 178} = 0{,}63$

$B = 0{,}63x + b$

$183{,}95 = 0{,}63 \cdot 265 + b$

$183{,}95 = 166{,}95 + b$

$17 = b$

$B = 0{,}63x + 17$

b. vastrecht: € 17,-

prijs per m³ water: € 0,63

c. $B = 0{,}63 \cdot 200 + 17 = 143$

d. $0{,}63x + 17 > 250$

$0{,}63x > 233$

$x > 369{,}8$

Dus ze verbruiken minstens 370 m³ water per jaar.

**Opgave 12:**

a. A: $K = 300 + 120 \cdot 1{,}4 = 468$

B: $K = 400 + 20 \cdot 2{,}4 = 448$

Dus bij 120 km kiezen ze voor maatschappij B

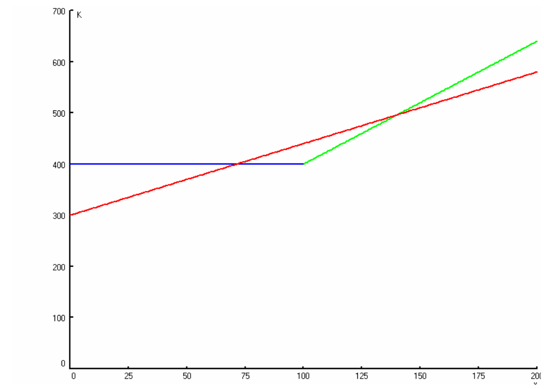
A: $K = 300 + 240 \cdot 1{,}4 = 636$

B: $K = 400 + 140 \cdot 2{,}4 = 736$

Dus bij 240 km kiezen ze voor maatschappij A

b. $K = 300 + 1{,}4x$

c.




d. $\begin{cases} K = 400 & 0 \le x \le 100 \\ K = 400 + 2{,}4(x - 100) & x \ge 100 \end{cases}$

e. $300 + 1{,}4x = 400$ en $300 + 1{,}4x = 400 + 2{,}4(x - 100)$

$1{,}4x = 100$

$x = 71{,}4$

$300 + 1{,}4x = 400 + 2{,}4x - 240$

$-x = -140$

$x = 140$

## Opgave 13:

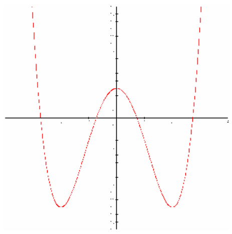
a. $y_1 = x^4 - 8x^2 + 4$

calc-menu optie minimum en maximum geeft:

min $f(-2) = -12$

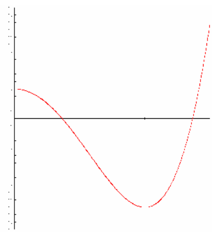
max $f(0) = 4$

min $f(2) = -12$



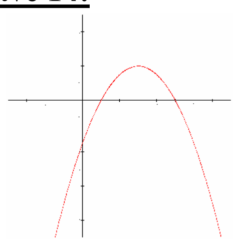
b. $f(3) = 13$

$B_f = [-12,13]$



## Opgave 14:

a.



b. $y_1 = -0{,}5x^2 + 3x - 2{,}5$

calc-menu optie maximum: $x = 3 \quad \wedge \quad y = 2$ dus max $f(3) = 2$

$B_f = \langle \leftarrow, 2]$

c. $T(3,2)$ en $C(0;-2{,}5)$

$rc = \frac{\Delta y}{\Delta x} = \frac{2--2{,}5}{3-0} = 1{,}5$

$y = 1{,}5x + b$ door $(0;-2{,}5)$

$-2{,}5 = b$

lijn $TC$: $y = 1{,}5x - 2{,}5$

d. $-0{,}5x^2 + 3x - 2{,}5 = 0$

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1 \quad \vee \quad x = 5$

$Opp(\Delta ABT) = \frac{1}{2} \cdot 4 \cdot 2 = 4$

**Opgave 15:**

a. $x_{top} = -\frac{b}{2a} = -\frac{4}{2} = -2$

$y_{top} = f(-2) = 4 - 8 + p = -4 + p$

$-4 + p < 1$

$p < 5$

b. $(-2,-4+p)$ op $y = 3x + 2$

$-4 + p = -6 + 2$

$p = 0$

**Opgave 16:**

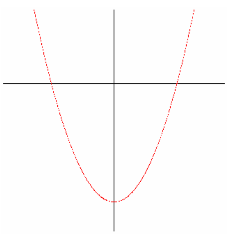
a. de grafiek is een dalparabool en heeft dan geen snijpunten met de $x$-as, dus $D < 0$.

$D = p^2 - 8 < 0$

$p^2 < 8$

$p = \sqrt{8} \quad \vee \quad p = -\sqrt{8}$

$-\sqrt{8} < x < \sqrt{8}$



b. $x_{top} = -\frac{b}{2a} = -\frac{p}{1} = -p$

$y_{top} = f(-p) = \frac{1}{2}(-p)^2 + p \cdot -p + 4 = \frac{1}{2}p^2 - p^2 + 4 = -\frac{1}{2}p^2 + 4$

$-\frac{1}{2}p^2 + 4 = -5$

$-\frac{1}{2}p^2 = -9$

$p^2 = 18$

$p = \sqrt{18} \quad \vee \quad p = -\sqrt{18}$

c. $-\frac{1}{2}p^2 + 4 = -3 \cdot -p + 8$

$-\frac{1}{2}p^2 + 4 = 3p + 8$

$-\frac{1}{2}p^2 - 3p - 4 = 0$

$p^2 + 6p + 8 = 0$

$(p+2)(p+4) = 0$

$p = -2 \quad \vee \quad p = -4$

**Opgave 17:**

$x_{top} = -\frac{b}{2a} = -\frac{2p}{2} = -p$ dus $p = -x_{top}$

$y_{top} = f(x_{top}) = (-p)^2 + 2p \cdot -p + \frac{5}{p} = p^2 - 2p^2 + \frac{5}{p} = -p^2 + \frac{5}{p}$

$y_{top} = -(-x_{top})^2 + \frac{5}{-x_{top}} = -(x_{top})^2 - \frac{5}{x_{top}}$

dus alle toppen liggen op de kromme: $y = -x^2 - \frac{5}{x}$

**Opgave 18:**

a. de top van *f* is het punt $(-2,4)$

$(-2,4) \xrightarrow{V_{x-as,2}} (-2,8) \xrightarrow{T(-2,-1)} (-4,7)$

b. $(-2,4) \xrightarrow{V_{x-as,4}} (-2,16) \xrightarrow{T(a,b)} (-2+a, 16+b)$

$-2 + a = 3$ dus $a = 5$

$16 + b = 5$ dus $b = -11$

c. $(-2,4) \xrightarrow{T(2,-6)} (0,-2) \xrightarrow{V_{x-as,c}} (0,-2c)$

**Opgave 19:**

a. $g(x) = 0{,}25(x+2)^2 - 4 + q$

$g(0) = 1 - 4 + q = 0$

$q = 3$

b. $h(x) = 0{,}25(x - p + 2)^2 - 4$

$h(0) = 0{,}25(-p+2)^2 - 4 = 0$

$0{,}25(-p+2)^2 = 4$

$(-p+2)^2 = 16$

$-p + 2 = 4 \quad \vee \quad -p + 2 = -4$

$-p = 2 \quad \vee \quad -p = -6$

$p = -2 \quad \vee \quad p = 6$

c. $B_f = [-4, \rightarrow\rangle$

$B_k = \langle\leftarrow, 6]$

$-4 \cdot a = 6$

$a = -1\frac{1}{2}$

d. $f(0) = -3$ en de top is $(-2,-4)$

$\begin{cases} m(-2) = 16a + b = -4 \\ m(0) = b = -3 \end{cases}$

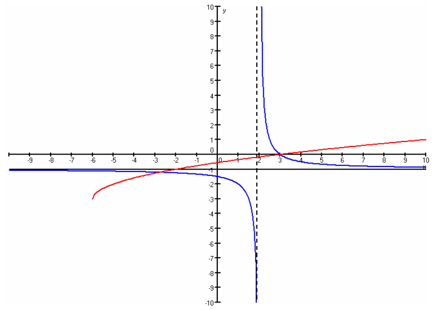
$16a - 3 = -4$

$16a = -1$

$a = -\frac{1}{16}$

**Opgave 20:**

a.



b. $y_1 = \sqrt{x+6} - 3$ en $y_2 = \dfrac{1}{x-2} - 1$

calc-menu intersection geeft: $x = -2{,}79 \quad \vee \quad x = 3$

$-6 \le x \le -2{,}79 \quad \vee \quad 2 < x \le 3$

c. $\sqrt{x+6} - 3 = 1$

$\sqrt{x+6} = 4$

$x + 6 = 16$

$x = 10$

d. $\dfrac{1}{x-2} - 1 = 5$

$\dfrac{1}{x-2} = 6$

$x - 2 = \frac{1}{6}$

$x = 2\frac{1}{6}$